Math 132
Homework 1 Comments

I graded 4 of the problems:
Section 1.1: 5
Section 1.2: 2ab
Section 1.3: 2
Section 1.4: 2bd

Each problem is worth 3 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 3 indicates a correct or nearly correct solution. Otherwise the grade given is 1 or 2 depending upon how much work was put in and how close the solution is to being correct. How neat and clear your solution is also affects the grade I give.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and return the homework in the next section.

The following are comments and occasionally solutions for the graded problems.

General Comments
The maximum number of points was 12. The high score was 11, and the median was 7.
I graded two algebraic problems and two graphing problems. They algebraic problems were essentially proofs using calculations, and I was disappointed that most people just wrote a string of equations with no explanation of what they were doing. Please include words describing what you are doing and what the logical flow is.

1.1 5. For some reason a lot of people did the proofs of $|\text{Re } z| \leq |z|$ and $|\text{Im } z| \leq |z|$ backwards—they assumed these held and then derived something true from them. This happened to be okay in this case because all the steps were reversible, but in general that doesn't always work so you need to be careful. A correct proof would be to write $z = x + iy$, and then note that $|\text{Re } z| = |x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} = |z|$; similarly $|\text{Im } z| = |y| = \sqrt{y^2} \leq \sqrt{x^2+y^2} = |z|$.

Most people had no trouble with the next part. There were several good approaches, including writing out z and w in terms of their real and imaginary parts and just calculating, but here's a nice simple proof: $|z+w|^2 = (z+w)\overline{(z+w)} = (z+w)(\bar{z}+\bar{w}) = z\bar{z} + w\bar{w} + z\bar{w} + w\bar{z} = |z|^2 + |w|^2 + z\bar{w} + \overline{z\bar{w}} = |z|^2 + |w|^2 + 2\text{Re } (z\bar{w})$, the last step following from the equality $\text{Re } z = (z+\bar{z})/2$ on page 2 of the text, which you can easily verify.

People had the most trouble with proving the triangle inequality from this. Most realized that $|z+w| \leq |z| + |w|$ if and only if $|z+w|^2 \leq (|z|+|w|)^2$ since everything is positive. They then expanded the right side to $|z|^2 + |w|^2 + 2|z||w|$ and realized it suffices to show $2\text{Re } (z\bar{w}) \leq 2|z||w|$, but this is where they ran into difficulty. The key here is to use what was proved at the beginning of the problem, and note that $|w| = |\bar{w}|$. So we have $2\text{Re } (z\bar{w}) \leq 2|\text{Re } (z\bar{w})| \leq 2|z\bar{w}| = 2|z||\bar{w}| = 2|z||w|$ and we're done.

1.2 2. I graded both parts (a)(b) assigned. It's very difficult to include figures in a Latex document so I won't try to draw them, but I did indicate a very rough sketch of the correct solution on the papers of those who made mistakes. For part (a) the set consits of the area between two rays of angle $\pi/4$ and $-\pi/4$. The most common mistakes were to include only the area between $\pi/4$ and angle 0, or to include the area between $\pi/4$ and $-3\pi/4$. Remember that negative for angles does not work quite the same way as negative for a point.

Part (b) was considerably harder and a lot of people had trouble with it. It seems the best way to proceed is to write $z = x + iy$ and then we have $z - 1 - i = (x-1) + i(y-1)$. Our inequality becomes $0 < \tan^{-1} \frac{y-1}{x-1} < \pi/3$. We can now take the tangent of everything, noting that tangent is continuous and increasing on $(0, \pi/3)$, and we get $0 < \frac{y-1}{x-1} < \sqrt{3}$. Geometric considerations rule out the case $x - 1 < 0$, so multiply everything by $x - 1$ to get $0 < y - 1 < \sqrt{3}(x-1)$, and so the area is the set of points satisfying both $y > 1$ and $y < \sqrt{3}x - \sqrt{3} + 1$. The problem can also be done purely geometrically.

1.3 2. I did this in section so most people did fine on this problem, and I graded it generously as well. The proof again is to let $P = (X, Y, Z)$, so $-P = (-X, -Y, -Z)$. Then if P projects to $z = x + iy$, we know that $x = \frac{X}{1-Z}$ and $y = \frac{Y}{1-Z}$. Similarly if $-P$ projects to $w = u + iv$ then $u = \frac{-X}{1+Z}$ and $v = \frac{-Y}{1+Z}$. We want to show that $w = -1/\bar{z}$; in other words that $\frac{-X}{1+Z} + i\frac{-Y}{1+Z} = -1/(\frac{X}{1-Z} - i\frac{Y}{1-Z})$. The right hand side equals $(Z-1)/(X-iY) = \frac{(Z-1)(X+iY)}{X^2+Y^2} = \frac{(Z-1)(X+iY)}{1-Z^2} = \frac{-X}{1+Z} + i\frac{-Y}{1+Z}$ as desired, where we have used the fact that $X^2 + Y^2 + Z^2 = 1$.

1.4 2. I did (c) in class, so I graded parts (b) and (d). Part (b) is similar to part (c). We want to find the set of points $u + iv$ such that $(u + iv)^2 = 1 + iy$ for some y. We get the pair of equations $u^2 - v^2 = 1$ and $2uv = y$; since y is arbitary the second equation is not a constraint and we are left with $u^2 - v^2 = 1$, the equation of a hyperbola. The principal branch is the right branch of the hyperbola and the other branch is the left branch. The asymptotes are the line $u = v$ and $u = -v$. This makes sense geometrically because the argument of points on the line $x = 1$ approaches $\pm\pi/2$ as y approaches $\pm\infty$, so the argument of the square root should approach $\pm\pi/4$.

For part (d) a similar approach can be tried although it is more complicated. Now we want the set of points $u + iv$ such that $(u + iv)^2 = x + i(x + 1)$ for some x. This gives the pair of equations $u^2 - v^2 = x$ and $2uv = x + 1$, which can be combined into one equation $2uv - u^2 + v^2 = 1$. So this also appears to look something like a hyperbola although I'm not sure how to treat it analytically. I found it easier to just graph some points and find out what the asymptotes should be. One complicating factor is that for the principal branch of square root, the negative real axis is not even part of the domain so the point where $y = x + 1$ crosses this axis does not map to any part of the range. Furthermore the graph actually splits at this point. Had we slit the plane on the positive real axis instead (a different branch of square root), the graph would be a continuous piece.

Now as $x \to \infty$ the argument goes to $\pi/4$, and as $x \to -1$ from the right the argument goes to π, so there is an asymptote at $\pi/8$ and the graph abruptly ends just before the point $(0, 1)$. In the other direction as $x \to -\infty$ the argument goes to $-3\pi/4$ and as $x \to -1$ from the left the argument goes to $-\pi/2$, so there is an asymptote at $-3\pi/8$ and the graph abruptly ends just before $(0, -1)$. This describes the principal branch (note that it is entirely in the right half plane as we expect), and the other branch is just the negative of this branch. Note that if we look at both branches at once, and fill in the two missing points at $(0, 1)$ and $(0, -1)$, we get a full graph of a hyperbola.