Math 132
Homework 2 Comments

I graded 4 of the problems:
Section 1.5: 2bc
Section 1.6: 2cd
Section 2.2: 3
Section 2.3: 3

Each problem is worth 3 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 3 indicates a correct or nearly correct solution. Otherwise the grade given is 1 or 2 depending upon how much work was put in and how close the solution is to being correct. How neat and clear your solution is also affects the grade I give.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and return the homework in the next section.

The following are comments and occasionally solutions for the graded problems.

General Comments

The maximum number of points was 12. The high score was 12, and the median was 8.
Again I graded two algebraic problems and two graphing problems. This problem set was overall easier than the previous one, and people did better. There was one 12, two 11's and several 10's, so that was nice to see. On the other hand there were also some very low scores. The next assignment is long and hard again, so make sure to start early.

1.5 2. I graded parts (b) and (c). Most people had no trouble with this. For part (b), the area is all points whose argument is between $\frac{5\pi}{3} = -\frac{\pi}{3}$ and $\frac{8\pi}{3} = \frac{2\pi}{3}$; that is to the right of the line formed by the rays of each of those two angles. Some people wrote the rays to look more like $-\frac{\pi}{6}$ and $\frac{5\pi}{6}$, but I didn't take off for this.

For part (b) the area consists of all points whose magnitude is between 1 and e, and whose argument is between 0 and $\frac{\pi}{4}$. The most common error was to include all points between those angles whose magnitude was between 0 and 1 or between 0 and e.

1.6 2. I graded parts (c) and (d). Again most people got this. The most common error was to forget that $-\pi < \operatorname{Arg} z \leq \pi$, so the imaginary part of $\operatorname{Log} z$ must lie in this range. So the unit circle of part (c) maps to the imaginary axis between the points $-i\pi$ and $i\pi$; some people said it was the entire imaginary axis. Similarly for part (d) the range is the rectangle of points whose real part is between $\frac{1}{2}$ and 2, and whose imaginary part is beween $-\pi$ and π.

2.2 3. Most people had a lot of trouble with this problem, although a couple people had very good solutions. This problem is a very good illustration of how complex differentiation differs from real differentiation, so it is important to understand it. The key point is that Δz is allowed to approach 0 from any direction in the complex plane, and the limit

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

must exist and give the same value for any such direction. For this problem the failure of $\bar{z}$ to be differentiable, described briefly as an example in the text, is a good one to follow.

Let me do the case $f(z) = \text{Re } z$. Just as in the $\bar{z}$ example, as well as the derivation of the Cauchy-Riemann equations, we can allow Δz to approach zero via two natural paths: along the positive real axis and along the positive imaginary axis. These are just two of infinitely many possible paths, but if we can show that the limit disagrees for just these two then it can't possibly exist. Now along the positive real axis we have $\Delta z = \epsilon$ for some real $\epsilon > 0$. We thus have

$$f'(z) = \lim_{\epsilon \to 0} \frac{\text{Re } (z + \epsilon) - \text{Re } (z)}{\epsilon} = \lim_{\epsilon \to 0} \frac{\epsilon}{\epsilon} = 1.$$

On the other hand along the positive imaginary axis we have $\Delta z = i\epsilon$ for some real $\epsilon > 0$ which results in

$$f'(z) = \lim_{i\epsilon \to 0} \frac{\text{Re } (z + i\epsilon) - \text{Re } (z)}{i\epsilon} = \lim_{i\epsilon \to 0} \frac{0}{i\epsilon} = 0.$$

Therefore Re z cannot be differentiable.

Now try $f(z) = \text{Im } z$ if you didn't already get the problem right, to make sure you understand how to do it.

2.3 3. I started this one in section, and people did much better on it than the previous problem. We have $f(z) = u(x,y) + iv(x,y)$ and $\bar{f}(z) = u(x,y) - iv(x,y)$, where $z = x + iy$. If both are analytic, then the Cauchy-Riemann equations are satisfied for both, which gives $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$, and also $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$. Since all of these must hold, we must also have $\frac{\partial v}{\partial y} = -\frac{\partial v}{\partial y}$, which implies $\frac{\partial v}{\partial y} = 0$. This then gives $\frac{\partial u}{\partial x} = 0$. Similarly $\frac{\partial v}{\partial x} = -\frac{\partial v}{\partial x}$ which implies $\frac{\partial v}{\partial x} = 0$, and thus also $\frac{\partial u}{\partial y} = 0$. So we have $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0$ (I didn't give full credit if you left out some of the partials), and thus both u and v must be constant on D since it is a domain. No one used the fact that D was a domain in their solution, but I didn't take off for this.