

Math 31B
Homework 6
Due Wednesday, February 21, 2007

Textbook Exercises to hand in

- **12.3:** 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 30, 38
- **12.4:** 4, 6, 8, 10, 12, 14, 18, 20, 30, 34, 37, 40, 42, 44, 46
- **Problems Plus (page 825):** 6

Additional Exercise to hand in

1. **Arc Length Contest Revisited.** Here is my entry for the arc length contest (p. 590): the curve that consists of a straight line from point $(0,0)$ to $(0,1)$, then $(0,1)$ to $(1,1)$, then $(1,1)$ to $(1,0)$.
 - (a) Draw a graph of my entry. It should be clear that the curve is continuous, has area 1 and intersects the x -axis at 0 and 1. Also the arc length is clearly 3, and so it beats most of the entries on page 590. However there is just one little problem. What is it?
 - (b) Let's try to fix my entry. For $n \geq 2$, define the function $f_n(x)$ as follows. For $0 \leq x \leq 1/n$, let $f_n(x) = \frac{n^2}{n-1}x$. For $1/n < x < (1 - 1/n)$, let $f_n(x) = \frac{n}{n-1}$. For $(1 - 1/n) \leq x \leq 1$, let $f_n(x) = \frac{n^2}{n-1}(1-x)$.
Draw graphs of $f_2(x)$, $f_3(x)$ and $f_{10}(x)$. For all $n \geq 2$, prove that $f_n(x)$ is continuous and also satisfies the three properties on page 590.
 - (c) Let a_n be the arc length of $f_n(x)$ for $n \geq 2$. Calculate a_n as a function of n . Let $a_1 = 5$ (does it matter what value we choose?) and examine the sequence $\{a_n\}_{n=1}^{\infty}$. Prove that the limit of this sequence is 3. What have we just shown?

Suggested warm-up exercises (do not hand these in)

- **12.3:** Selected odd exercises from 3 to 23; 31, 33
- **12.4:** Selected odd exercises from 3 to 45