Math 32AH
Homework 3 Solutions

I graded 4 of the problems:
Page 110: 6, 8, 13, 14.

Each problem is worth 3 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 3 indicates a correct or nearly correct solution. Otherwise the grade given is 1 or 2 depending upon how much work was put in and how close the solution is to being correct. How neat and clear your solution is also affects the grade I give.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and return the homework in the next section.

The following are solutions to the homework problems and additional comments for the problems I graded.

General Comments
The maximum number of points was 12. The high score was 12, the median was 12 and the mean was 11.2. This was an easy computational assignment and almost everyone got 11 or 12 on it. A couple people asked about the relationship between a function in "implicit" form $y = f(x)$ and "parameterized" form $\mathbf{x}(t) = (x(t), y(t))$; this is a good point to make sure you understand so I'll try to talk about it in section, or feel free to ask about it in office hours.

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1. Solution is in the back of the book.

6. We have $\mathbf{x}'(t) = (e^t(\cos t - \sin t), e^t(\cos t + \sin t), e^t)$ and $|\mathbf{x}'(t)| = \sqrt{3}e^t$, so $\mathbf{T} = \frac{1}{\sqrt{3}}(\cos t - \sin t, \cos t + \sin t, 1)$. Then $\mathbf{N} = \frac{1}{\sqrt{2}}(-\sin t - \cos t, \cos t - \sin t, 0)$ and $K = \frac{\sqrt{2}}{3e^t}$.

7. Solution is in the back of the book.

8. We have $\mathbf{x}'(t) = (e^t(\cos t - \sin t), e^t(\cos t + \sin t), 0)$, $|\mathbf{x}'(t)| = \sqrt{2}e^t$, and $\mathbf{x}''(t) = \mathbf{a}(t) = (-2e^t \sin t, 2e^t \cos t, 0)$. Therefore $\mathbf{T} = \frac{1}{\sqrt{2}}(\cos t - \sin t, \cos t + \sin t, 0)$. It follows $\mathbf{a_T} = (\mathbf{a} \cdot \mathbf{T})\mathbf{T} = \sqrt{2}e^t\mathbf{T} = (e^t(\cos t - \sin t), e^t(\cos t + \sin t), 0)$. It then follows $\mathbf{a_N} = \mathbf{a} - \mathbf{a_T} = (-e^t \sin t - e^t \cos t, e^t \cos t - e^t \sin t, 0)$.

13. We have $\mathbf{v}(t) = \mathbf{x}'(t) = (x'(t), y'(t))$, and $\mathbf{a}(t) = (x''(t), y''(t))$. Using Theorem 3.2, we calculate $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \left(\frac{x'(t)}{\sqrt{x'(t)^2+y'(t)^2}}, \frac{y'(t)}{\sqrt{x'(t)^2+y'(t)^2}}\right)$. Therefore $\mathbf{T}'(t) = \left(\frac{\sqrt{x'^2+y'^2}x''-x'(x'^2+y'^2)^{-1/2}(x''+y'')}{x'^2+y'^2}, \frac{\sqrt{x'^2+y'^2}y''-y'(x'^2+y'^2)^{-1/2}(x''+y'')}{x'^2+y'^2}\right) =$ $\left(\frac{(x'^2+y'^2)x''-x'(x''+y'')}{(x'^2+y'^2)^{3/2}}, \frac{(x'^2+y'^2)y''-y'(x''+y'')}{(x'^2+y'^2)^{3/2}}\right)$. If I haven't made a mistake then calculating the length of this and then dividing by $|\mathbf{x}'(t)| = \sqrt{x'^2 + y'^2}$ should give the answer.

    A much easier way to solve this problem is to use Theorem 3.10, which gives the answer with almost no work. However it doesn't apply immediately because the theorem holds for curves in $\mathbb{R}^3$ whereas our curve is only in $\mathbb{R}^2$. However we can "embed" the curve in $\mathbb{R}^3$ by imagining it to be in the plane $z = 0$. So now we have $\mathbf{x}(t) = ((x(t), y(t), 0)$, $\mathbf{v}(t) = (x'(t), y'(t), 0)$, and $\mathbf{a}(t) = (x''(t), y''(t), 0)$. It seems clear that the curvature of our curve should be the same in this case. And indeed it is; if you plug into the equation of Theorem 3.10 you'll get the result immedately.

14. Following the hint, note that one can parameterize the curve $y = f(x)$ as $\mathbf{x}(t) = (x(t), y(t)) = (t, f(t))$. Now plug into the formula derived in exercise 13 to immediately get the answer.