# Math 32AH
# Homework 6 Solutions

I graded 4 of the problems:
Page 153: 8;
Page 171: 9, 17, 21.

Each problem is worth 3 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 3 indicates a correct or nearly correct solution. Otherwise the grade given is 1 or 2 depending upon how much work was put in and how close the solution is to being correct. How neat and clear your solution is also affects the grade I give.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and return the homework in the next section.

The following are solutions to the homework problems and additional comments for the problems I graded.

**General Comments**
The maximum number of points was 12. The high score was 12, the median was 12 and the mean was 10.8. I thought this homework might be a little more challenging than the last two so I'm happy to see everyone rose to the challenge!

**Page 153**

2. $\frac{\partial f}{\partial x} = \frac{-x}{\sqrt{4-x^2-y^2}} = \frac{-1}{\sqrt{2}}$ at $(1,1)$. $\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{4-x^2-y^2}} = \frac{-1}{\sqrt{2}}$ at $(1,1)$.
5. Solution is in the back of the book.
8. The normal vector to the tangent plane is $\mathbf{n} = (-1, -1/2, -1)$, and we can use a multiple so let $\mathbf{n} = (2,1,2)$. Then an equation of the tangent plane is $(2,1,2)\cdot(x-2, y-1, z-2) = 0$ or $(2x-4)+(y-1)+(2z-4) = 0$ or $2x+y+2z = 9$.
9. Solution is in the back of the book.
13. Solution is in the back of the book.
15. Solution is in the back of the book.

**Page 160**

1. Nonlinear. For example $0 = 3\sin(3\pi) \neq 3\sin(\pi/2) + 3\sin(5\pi/2) = 6$.
4. Nonlinear (but affine). For example $10 + 4x = 2(5+2x) \neq 5 + [2(2x)] = 5 + 4x$ for any $x$.
12. Nonlinear (although linear in each variable if the other variables are fixed). For example $f(1,1,1) = 1+2 = 3$ and $f(2,2,2) = 4+4 = 8$ which is not twice $f(1,1,1)$.

**Page 171**

2. We have $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) = -y\sin xy, -x\sin xy)$ and these are clearly continuous so this is the total derivative. Plugging in, we have $f'(1, \frac{\pi}{3}) = (-\frac{\pi}{3}\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}) = (-\frac{\pi}{2\sqrt{3}}, -\frac{\sqrt{3}}{2})$.
5. Solution is in the back of the book.
9. We first determine what $\ell$ should be by taking partial derivatives. We have $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) = (2x, 2y)$, so at the point $(2,1)$ this is $(4,2)$. So let $\ell(\mathbf{x}) = (4,2)\cdot\mathbf{x}$. We want to prove that $f(x,y) = f(2,1) + (4,2)\cdot(x-2, y-1) + e(x,y)$, where $\lim_{(x,y)\to(2,1)} \frac{e(x,y)}{|(x-2,y-1)|} = 0$. Using the definition of $f$, this means $x^2+y^2 = 5 + 4x + 2y - 10 + e(x,y)$, which means $e(x,y) = (x^2-4x+4) + (y^2-2y+1) = (x-2)^2 + (y-1)^2$. Finally we have $\lim_{(x,y)\to(2,1)} \frac{e(x,y)}{|(x-2,y-1)|} = \lim_{(x,y)\to(2,1)} \frac{(x-2)^2+(y-1)^2}{\sqrt{(x-2)^2+(y-1)^2}} = \lim_{(x,y)\to(2,1)} \sqrt{(x-2)^2+(y-1)^2} = 0$, as desired.

11. As in the hint we use $f(x,y) = \sqrt{x^2+y^2}$, and it follows that $f(3.01, 3.98) = f(3+.01, 4-.02) \approx f(3,4) + \left(\frac{\partial f}{\partial x}(3,4), \frac{\partial f}{\partial y}(3,4)\right) \cdot (.01, -.02) = 5 + (\frac{3}{5}, \frac{4}{5}) \cdot (.01, -.02) = 5 + .006 - .016 = 4.99$.

17. As in problem 11 we use $f(x,y,z) = \sqrt{x^2+y^2+z^2}$, and it follows that $f(2.01, 1.94, 0.98) = f(2+.01, 2-.06, 1-.02) \approx f(2,2,1) + \left(\frac{\partial f}{\partial x}(2,2,1), \frac{\partial f}{\partial y}(2,2,1), \frac{\partial f}{\partial z}(2,2,1)\right) \cdot (.01, -.06, -.02) = 3 + (\frac{2}{3}, \frac{2}{3}, \frac{1}{3}) \cdot (.01, -.06, -.02) = 3 - \frac{2}{3}\frac{6}{100} = 2.96$.

21. We can use our usual techniques to show this function is not continuous at $(0,0)$ and therefore cannot be differentiable there. For example approach $(0,0)$ via the line $y = x$, so $\lim_{x\to 0} \frac{3x^2}{2x^2} = \frac{3}{2} \neq 0$.